CONNECTION BETWEEN THE CHARACTERISTICS OF THE SIGNAL OF A SHADOW INSTRUMENT AND THE SPECTRUM OF THE TURBULENCE

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The shadow method is one of the most widely used methods for investigating turbulence [1, 2]. Using a shadow instrument with photoelectric recording, there exists, in principal, the possibility of finding the statistical characteristics of the turbulence from the statistical characteristics of the random signal taken from the instrument. In the present work, an investigation is made of the connection between the mean value and the scattering of the signal of a shadow instrument and the energy spectrum of optical inhomogeneities in the medium.

1. Connection between the Mean Value and the Scattering of the Signal and the Moments of the Light Field

We consider the overall scheme of the shadow instrument illustrated in Fig. 1. A coherent monochromatic light beam from the illuminator 1 passes through a layer of the medium under investigation, with a thickness L, located between the planes 2 and 3. The plane 3 is the front focal plane of the lens 5; a shadow diaphragm is located at its rear focal plane 4 (the shadow plane). The light passing through the shadow plane is collected by lens 6 to the photoelectronic multiplier 7. In what follows, by the "signal of the instrument" we shall understand the intensity of the light falling on the photoelectronic multiplier.

We introduce the Cartesian coordinates x, y, z in such a way that the z axis will be directed along the axis of the propagation of the light; the plane 2 corresponds to $z=0$, and the plane 3 to $z=L$. Let $u(x, y, L) \equiv u(\mathbf{x})$, $\mathbf{x}=(x, y)$ be the random distribution of the field at the plane 3. Then in the plane 4 the distribution of the field in the coordinates $\varkappa=(\varkappa_1, \varkappa_2)$, connected with coordinates $\mathbf{x}=(x, y)$ by the relationship $\varkappa=(2\pi/\lambda f)\mathbf{x}$ (λ is the wavelength of the light; f is the focal distance of lens 5), is a Fourier transform of $u(\mathbf{x})$. In the case where the distance between the plane 3 and the lens 5 is not equal to the focal distance of the lens, the field in the plane 4 differs from the Fourier transform of the field u by a factor, equal to unity in its modulus [3].

We denote by E the energy of the light passing through the shadow diaphragm, collected by lens 6, and sent to the photoreceiver 7. Then for the mean value of the signal of the shadow instrument $\langle E\rangle$ (the angular brackets mean averaging with respect to the ensemble of realizations of the random medium) we obtain the expression

$$\langle E\rangle=\frac{1}{(2\pi)^2}\int d\mathbf{x}_1\int d\mathbf{x}_2\int d\varkappa\,\Gamma(\mathbf{x}_1, \mathbf{x}_2; L)\,e^{i\varkappa(\mathbf{x}_1-\mathbf{x}_2)}\chi(\varkappa),$$

where the quantity $\Gamma(\mathbf{x}_1, \mathbf{x}_2; L) \equiv \langle u(\mathbf{x}_1)\bar{u}(\mathbf{x}_2)\rangle$ is a function of the mutual coherence [4] or the second moment of the random field u in the plane $z=L$; $\chi(\varkappa)$ is a transmission function with respect to the intensity of the shadow diagram. Analogously to (1.1), an expression can be written for the scattering D of the signal:

$$D\equiv\langle(E-\langle E\rangle)^2\rangle=\frac{1}{(2\pi)^2}\int d\mathbf{x}_1\int d\mathbf{x}_2\int d\mathbf{x}_3\int d\mathbf{x}_4\int d\varkappa_1\int d\varkappa_2\,\chi\times \tag{1.2}$$

$$\times(\varkappa_1)\chi(\varkappa_2)\,e^{i\varkappa_1(\mathbf{x}_1-\mathbf{x}_2)}e^{i\varkappa_2(\mathbf{x}_3-\mathbf{x}_4)}\left(\Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L)-\Gamma(\mathbf{x}_1, \mathbf{x}_2; L)\,\Gamma(\mathbf{x}_3, \mathbf{x}_4; L)\right),$$

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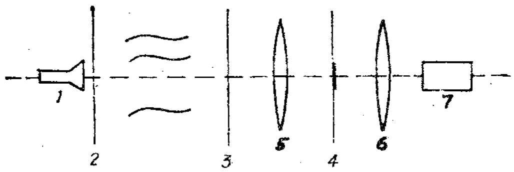

Fig. 1

where the quantity $\Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L) \equiv \langle u(\mathbf{x}_1)\bar{u}(\mathbf{x}_2)u(\mathbf{x}_3)\bar{u}(\mathbf{x}_4)\rangle$ is the fourth moment of the field u in the plane $z = L$, and $\hat{\Gamma}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L) \equiv \Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L) - \Gamma(\mathbf{x}_1, \mathbf{x}_2; L)\Gamma(\mathbf{x}_3, \mathbf{x}_4; L)$ is the centered fourth moment.

2. Calculation of the Moments of the Light Field in the Born Approximation

We use the following assumptions with respect to the random medium. The field of the dielectric constant $\varepsilon(\mathbf{r})$, $\mathbf{r} = (x, y, z)$ is assumed to be statistically homogeneous and isotropic. The dimension of the smallest homogeneities in the medium is assumed to be much less than the wavelength of the light, which makes it possible to use a scalar theory of propagation [5]. The fluctuations of the dielectric constant are assumed to be small:

$$\varepsilon(\mathbf{r}) = \langle\varepsilon\rangle(1 + \varepsilon'\mathbf{r})), \; |\varepsilon'(\mathbf{r})| \ll 1, \tag{2.1}$$

where $\langle\varepsilon\rangle$ (the mean value of the dielectric constant) does not depend on the coordinates, by virtue of the assumptions adopted.

The field $u(\mathbf{r})$ satisfies the boundary-value problem

$$\begin{cases} \Delta u(\mathbf{r}) + k^2(1 + \varepsilon'(\mathbf{r}))\, u(\mathbf{r}) = 0, \\ u(\mathbf{r})|_{z=0} = u_0(\mathbf{x}), \\ \lim\limits_{r\to\infty} r\left(\dfrac{\partial}{\partial r} - ik\right) u = 0, \end{cases} \tag{2.2}$$

where $\mathbf{r} = (x, y, z)$; $\mathbf{x} = (x, y)$; $k^2 = (2\pi)^2/\lambda^2 \langle\varepsilon\rangle$; λ is the wavelength of the light. By virtue of (2.1), the solution of the problem (2.2) can be sought in the form of a Born expansion [6]:

$$u(\mathbf{r}) = V_0(\mathbf{r}) + V_1(\mathbf{r}) + V_2(\mathbf{r}) + \ldots, \tag{2.3}$$

where $V_i(\mathbf{r})$, $i = 0, 1, 2, \ldots$, has the i-th order with respect to the value of ε'. Substituting (2.3) with $\mathbf{r} = (x, y, L)$ into the determinations of the second and fourth moments, and neglecting terms proportional to powers of ε' higher than the second, we obtain

$$\Gamma(\mathbf{x}_1, \mathbf{x}_2; L) = \sum_{i,j=0}^{i+j=2} \Gamma_{ij}(\mathbf{x}_1, \mathbf{x}_2); \tag{2.4}$$

$$\Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L) = \sum_{i,j,k,l=0}^{i+j+k+l=2} \Gamma_{ijkl}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4), \tag{2.5}$$

where $\mathbf{x}_m = (x_m, y_m)$, $m = 1, 2, 3, 4$;

$$\Gamma_{ij}(\mathbf{x}_1, \mathbf{x}_2) \equiv \langle V_i(x_1, y_1 L) < \bar{V}_j(x_2, y_2, L)\rangle; \tag{2.6}$$

$$\Gamma_{ijkl}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \equiv \langle V_i(x_1, y_1, L)\, \bar{V}_j(x_2, y_2, L)\, V_k(x_3, y_3, L)\, \bar{V}_l(x_4, y_4, L)\rangle. \tag{2.7}$$

Analogously,

$$\Gamma(\mathbf{x}_1, \mathbf{x}_2; L)\,\Gamma(\mathbf{x}_3, \mathbf{x}_4; L) = \sum_{i,j,k,l=0}^{i+j+k+l=2} \Gamma_{ij}(\mathbf{x}_1, \mathbf{x}_2)\,\Gamma_{kl}(\mathbf{x}_3, \mathbf{x}_4). \tag{2.8}$$

Taking into consideration that $\Gamma_{00}(\mathbf{x}_1, \mathbf{x}_2)\Gamma_{ij}(\mathbf{x}_3, \mathbf{x}_4) = \Gamma_{00ij}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$ and $\Gamma_{ij}(\mathbf{x}_1, \mathbf{x}_2)\Gamma_{00}(\mathbf{x}_3, \mathbf{x}_4) = \Gamma_{ij00}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$, from (2.5), (2.8) we obtain an expression for the centered fourth moment:

$$\hat{\Gamma}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L) = \Gamma_{1001}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) + \Gamma_{0110}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) + \Gamma_{1010}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)\Gamma_{0101}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4). \tag{2.9}$$

To calculate the terms entering into (2.4), (2.9), we must find $V_0(\mathbf{r})$ and $V_1(\mathbf{r})$ in the plane $z = L$. The field of $u_0(\mathbf{x})$ in the plane $z = 0$ is given in the form

$$u_0(\mathbf{x}) = A \exp\left(-\frac{\mathbf{x}^2}{2a^2}\right). \tag{2.10}$$

Substituting into (2.2) u(r) in the form of the expansion (2.3), by the usual method we find

$$V_0(\mathbf{r}) = \frac{A}{B(z)} \exp\left(-\bar{B}(z)\frac{x^2+y^2}{2a^2|B(z)|^2} + ikz\right), \tag{2.11}$$

where $B(z) \equiv 1 + i(z/ka^2)$;

$$V_1(\mathbf{r}) = -\frac{k^2}{4\pi}\int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\varepsilon'(\mathbf{r}')V_0(\mathbf{r}')\,d\mathbf{r}'. \tag{2.12}$$

We use the "sagittal approximation" for the Green's function in (2.12), and assuming that $|\mathbf{x}-\mathbf{x}'|^2/(z-z')^2 \ll 1$ ($\mathbf{x} = (x, y)$), we write

$$\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} \simeq \frac{\exp\left(|z-z'| + \frac{|\mathbf{x}-\mathbf{x}'|^2}{|z-z'|}\right)}{|z-z'|}.$$

We shall use also the following usual assumptions [5]:

$$kl \gg 1; \tag{2.13}$$

$$ka \gg 1; \tag{2.14}$$

$$l \ll L, \tag{2.15}$$

where l is the internal scale of the inhomogeneities.

3. Mean Value of Signal of Shadow Instrument

Assuming in (2.11), (2.12) that $\mathbf{r} = (x, y, L)$, and substituting into (2.6), after certain transformations, taking account of the assumptions made, we obtain the following terms in the right-hand part of (2.4):

$$\Gamma_{00}(\mathbf{x}_1, \mathbf{x}_2) = \frac{A^2}{|B(L)|^2}\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{2a^2|B(L)|^2} + i\frac{L}{2ka^4}\frac{\mathbf{x}_1^2-\mathbf{x}_2^2}{|B(L)|^2}\right); \tag{3.1}$$

$$\Gamma_{10}(\mathbf{x}_1, \mathbf{x}_2) = \Gamma_{01}(\mathbf{x}_1, \mathbf{x}_2) = 0; \tag{3.2}$$

$$\Gamma_{20}(\mathbf{x}_1, \mathbf{x}_2) = \bar{\Gamma}_{02}(\mathbf{x}_1, \mathbf{x}_2) = -\frac{k^2LA^2}{4|B(L)|^2}\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{2a^2|B(L)|^2} + i\frac{L}{2ka^4}\frac{\mathbf{x}_1^2-\mathbf{x}_2^2}{|B(L)|^2}\right)\int_0^\infty \sigma(s)\,ds, \tag{3.3}$$

where $\sigma(|\mathbf{r}''-\mathbf{r}'|) \equiv \sigma(\mathbf{r}'',\mathbf{r}') \equiv \langle\varepsilon'(\mathbf{r}')\varepsilon'(\mathbf{r}'')\rangle$ is the correlation function of the fluctuations of the dielectric constant;

$$\Gamma_{11}(\mathbf{x}_1, \mathbf{x}_2) = \frac{k^2A^2}{4|B(L)|^2}\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{2a^2|B(L)|^2} + i\frac{L}{2ka^4}\frac{\mathbf{x}_1^2-\mathbf{x}_2^2}{|B(L)|^2}\right)\times$$
$$\times\int_0^L dp\int d\eta\,\Phi(\eta)\exp\left(-\eta^2\frac{(L-p)^2}{k^2a^2|B(L)|^2} + i\beta(p)\mathbf{x}_1\eta - i\bar{\beta}(p)\mathbf{x}_2\eta\right), \tag{3.4}$$

where

$$\beta(p) \equiv \frac{B(p)}{B(L)};\quad \Phi(\eta) \equiv \Phi(\eta) = \frac{1}{(2\pi)^2}\int\int\int_{-\infty}^{\infty}\sigma(|\mathbf{r}|)\,e^{i\eta_1x+i\eta_2y}\,dx\,dy\,dz;$$

$$\eta = (\eta_1, \eta_2)$$

is a two-dimensional Fourier transform of the correlation function. [It can be shown that, in the case of the one-dimensional isotropic field under consideration, the two-dimensional spectrum of $\Phi(\eta)$ is connected with the three-dimensional Fourier transform of the correlation function $F(\eta)$ by the relationship $\Phi(\eta) = 2\pi F(\eta)$.]

Substituting into (1.1) the expression for the second moment (2.4), we obtain

$$\langle E\rangle = I_{00} + I_{20} + I_{02} + I_{11},$$

where

$$I_{ij} \equiv \frac{1}{(2\pi)^2}\int\int\int \Gamma_{ij}(\mathbf{x}_1, \mathbf{x}_2)\,e^{i\varkappa(\mathbf{x}_1-\mathbf{x}_2)}\chi(\varkappa)\,d\mathbf{x}_1 d\mathbf{x}_2 d\varkappa. \tag{3.5}$$

Calculation of (3.5) with the functions $\Gamma_{ij}(\mathbf{x}_1, \mathbf{x}_2)$ from (3.1)-(3.4) leads to the result

$$I_{00} = A^2a^4\int \chi(\varkappa)\,e^{-\varkappa^2a^2}\,d\varkappa;$$

$$I_{20} = I_{02} = -\frac{\pi k^2 L A^2 a^4}{4} \int_0^\infty \Phi(\eta)\,\eta\, d\eta \int \chi(\varkappa)\, e^{-\varkappa^2 a^2} d\varkappa; \tag{3.6}$$

$$I_{11} = \frac{k^2 L A^2 a^4}{2} \int_0^\infty \Phi(\eta)\,\eta \left\{ \int \chi(\varkappa)\, e^{-a^2(\eta^2+\varkappa^2)} I_0(2a^2\eta\varkappa)\, d\varkappa \right\} d\eta, \tag{3.7}$$

where $I_0(z)$ is a Bessel function of an imaginary argument with a zero subscript.

The quantity I_{00} is the value of the signal in the absence of inhomogeneities (the "background illumination"); in what follows, we shall consider the mean value of the deviation of the signal of the shadow instrument from the background value I_{00}, i.e., $E' = E - I_{00}$. From (3.6), (3.7) it follows that

$$\langle E' \rangle = \frac{\pi a^4 A^2 k^2 L}{2} \int_0^\infty \Phi(\eta)\,\eta \left\{ \int \chi(\varkappa)\, e^{-a^2\varkappa^2} \left[e^{-a^2\eta^2} I_0(2a^2\eta\varkappa) - 1 \right] d\varkappa \right\} d\eta. \tag{3.8}$$

4. Scattering of Signal of Shadow Instrument

Setting $\mathbf{r} = (x, y, z)$ in (2.11), (2.12) and substituting into (2.7), after certain transformations taking account of (2.13)-(2.15), we obtain the following terms in the right-hand part of (2.9):

$$\Gamma_{1001}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = \Gamma_{0000}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \frac{k^2}{4} \int_0^L dp \int d\eta \Phi(\eta) \exp\left[-\eta^2 \frac{(L-p)^2}{k^2 a^2 |B(L)|^2} + i\beta(p)\,\mathbf{x}_1\eta - i\bar{\beta}(p)\,\mathbf{x}_4\eta \right); \tag{4.1}$$

$$\Gamma_{0110}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = \bar{\Gamma}_{1001}(\mathbf{x}_2, \mathbf{x}_1, \mathbf{x}_4, \mathbf{x}_3); \tag{4.2}$$

$$\Gamma_{1010}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = -\Gamma_{0000}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \frac{k^2}{4} \int_0^L dp \int d\eta \Phi(\eta) \tag{4.3}$$

$$\times \exp\left[-\eta^2 \frac{(L-p)^2}{k^2 a^2 |B(L)|^2} - i\eta^2 \left(1 + \frac{pL}{k^2 a^4}\right) \frac{L-p}{k|B(L)|^2} + i\beta(p)\mathbf{x}_1\eta - i\beta(p)\,\mathbf{x}_3\eta \right];$$

$$\Gamma_{0101}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = \bar{\Gamma}_{1010}(\mathbf{x}_2, \mathbf{x}_1, \mathbf{x}_4, \mathbf{x}_3). \tag{4.4}$$

Substituting expressions (4.1)-(4.4) into (2.9), we obtain the centered fourth moment $\Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; L)$. Now, from (1.2) we have

$$D = 2\pi k^2 A^4 a^8 \int_0^\infty \Phi(\eta)\, e^{-a^2\eta^2} \eta \left\{ \int_0^L dp \left[\cos\frac{p\eta^2}{2k} \operatorname{Im}\psi(p,\eta) - \sin\frac{p\eta^2}{2k} \operatorname{Re}\psi(p,\eta) \right]^2 \right\} d\eta, \tag{4.5}$$

where

$$\psi(p,\eta) = \int \chi(\varkappa)\, e^{-a^2\varkappa^2} J_0\left(ia^2 \left[1 + i\frac{p}{ka^2} \right] \eta\varkappa \right) d\varkappa; \tag{4.6}$$

$J_0(z)$ is a Bessel function of zero subscript.

5. Analysis of Results

As a shadow diaphragm, in practice, diaphragms of one of the following two types are used: 1) a shadow diaphragm in the form of an opaque half-plane; the corresponding transmission function is

$$\chi_p(\varkappa) = \begin{cases} 1 & \varkappa_1 > 0, \\ 0 & \varkappa_1 \leqslant 0, \end{cases} \qquad \varkappa = (\varkappa_1, \varkappa_2);$$

2) a shadow diaphragm in the form of an opaque circle; the transmission function is

$$\chi_c(\varkappa) = \begin{cases} 1 & \varkappa > \frac{1}{a}, \\ 0 & \varkappa \leqslant \frac{1}{a}. \end{cases} \tag{5.1}$$

The selection of the radius of the circle in (5.1) is dictated by the distribution of the intensity of the light in the shadow plane in the case of the absence of inhomogeneities in the volume under investigation; for the light beam (2.10), this distribution has the form $A_1 \exp(-a^2\varkappa^2)$, where A_1 is a vector constant.

We limit ourselves to an analysis of the results obtained (3.8), (4.5), and (4.6) for the case of a "Gaussian" shadow diagram with the transmission function

$$\chi_\Gamma(\varkappa) = 1 - e^{-a^2\varkappa^2}. \tag{5.2}$$

The results for such a diaphragm are found to be qualitatively close to results calculated for a round diaphragm (5.1); however, the transmission function (5.2) considerably simplifies the analysis.

Substitution of (5.2) into (3.8) gives

$$\langle E'\rangle = \frac{\pi k^2 L}{4}\{\pi a^2 A^2\}\int_0^\infty \Phi(\eta)\,\eta\left[1-\exp\left(-\frac{a^2\eta^2}{2}\right)\right]d\eta. \tag{5.3}$$

The quantity in shaped brackets in (5.3) is equal to the energy of the light beam, integrated over the transverse cross section of the intensity.

The energy spectrum $\Phi(\eta)$ in the inertial interval of frequencies falls with a rise in η; for the Kolmogorov spectrum $(\Phi(\eta) \sim \eta^{-11/3}$ [5]. Thus, in the inertial interval of frequencies, the function under the integral sign in (5.3) has the form (with an accuracy up to a factor) $\eta^{-8/3}[1-\exp(-a^2\eta^2/2)]$. This function falls monotonically (as $\eta^{-2/3}$ with $\eta \ll a^{-1}$, and $\eta^{-8/3}$ with $\eta \gg a^{-1}$). Consequently, the greatest contribution to the value of the mean value of the signal is that of the inhomogeneities with the lowest spatial frequencies. Small-scale inhomogeneities have only a slight effect on the mean value of the signal of the shadow instrument.

With substitution of (5.2) into (4.5), (4.6), we obtain

$$D = \frac{\pi k^2}{2}\{\pi^2 a^4 A^4\}\int_0^\infty \Phi(\eta)\exp\left(-\frac{3a^2\eta^2}{4}\right)\eta\left[\int_0^L \exp\left(-\frac{\eta^2 p^2}{4k^2a^2}\right)\sin^2\left(\frac{p\eta^2}{4k}\right)dp\right]d\eta. \tag{5.4}$$

The quantity in shaped brackets is equal to the square of the energy of the light beam.

We introduce the following two conditions:

$$\left(\frac{L\lambda}{l^2}\right)^2 \ll 1;\quad \left(\frac{L\lambda}{al}\right)^2 \ll 1. \tag{5.5}$$

The second condition follows from the first with satisfaction of the usual relationship $a \geq l$; the first is a weakened form of the usual condition of the smallness of the diffraction effects $\sqrt{\lambda L} \ll l$. From (5.5) there follow the relationships

$$\left(\frac{p\eta^2}{4k}\right)^2 \leqslant \left(\frac{L}{kl^2}\right)^2 = 0\left[\left(\frac{L\lambda}{l^2}\right)^2\right] \ll 1; \tag{5.6}$$

$$\left(\frac{p\eta}{ka}\right)^2 \leqslant \left(\frac{L}{lka}\right)^2 = 0\left[\left(\frac{L\lambda}{al}\right)^2\right] \ll 1. \tag{5.7}$$

Taking (5.6), (5.7) into consideration, in the expansion in series of the expression under the integral sign in the integral with respect to p in (5.4), we can limit ourselves to the first term,

$$\exp\left(-\frac{\eta^2 p^2}{4k^2a^2}\right)\sin^2\left(\frac{p\eta^2}{4k}\right) \simeq \frac{1}{16}\frac{p^2\eta^4}{k^2}.$$

Now, from (5.4) we have

$$D = \frac{\pi L^3}{96}\{\pi^2 A^4 a^4\}\int_0^\infty \Phi(\eta)\exp\left(-\frac{3a^2\eta^2}{4}\right)\eta^5 d\eta. \tag{5.8}$$

We note that the weighting function $\exp[-(3a^2\eta^2/4)]\eta^5$ in (5.8) has a maximum with $\eta_* = \sqrt{(10/3)}\,a^{-1} \simeq 1.8a^{-1}$, whose "sharpness" can be judged from the "half-width" Δ,

$$\Delta \equiv \frac{\int_0^\infty \exp\left(-\frac{3a^2\eta^2}{4}\right)\eta^5 d\eta}{\exp\left(-\frac{3a^2\eta_*^2}{4}\right)\eta_*^5} \simeq 1.4a^{-1}.$$

The decrease in the energy spectrum $\Phi(\eta)$ in the inertial interval ($\sim\eta^{-11/3}$ for the Kolmogorov spectrum) screens somewhat the action of the factor η^5 in (5.8), weakening the effect of large inhomogeneities (the low-frequency part of the spectrum). Thus, we note the following:

a) the greatest contribution to the scattering of the signal of a shadow instrument of the type under consideration is that of optical inhomogeneities with spatial frequencies close to $\eta_* \simeq 1.8a^{-1}$ (i.e., inhomogeneities with dimensions close to $l_* = (2\pi/\eta_*) \simeq 3.4a$), where a is the radius of the light beam coming out of the illuminator;

b) small inhomogeneities, whose spatial frequencies are greater than $\eta_* + \Delta \simeq 3.2a^{-1}$, make no contribution to the value of the scattering of the signal of the instrument;

c) inhomogeneities, whose spatial frequencies are small in comparison with $\eta_* \simeq 1.8a^{-1}$, do not, in practice, have any effect on the scattering of the signal.

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LAG TIME OF THE BREAKDOWN OF PRESSED LEAD AZIDE

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Data are obtained on the lag time of the breakdown and the dielectric strength of a solid porous dielectric with a porosity of 0.4 (pressed lead azide) with different durations of a rectangular voltage pulse from 10^{-8} to $2 \cdot 10^{-6}$ sec.

Important characteristics of the pulsed breakdown of dielectrics are the lag time of the breakdown and the dielectric strength. A lag of the breakdown is observed both for gases and for solid dielectrics. It consists of the statistical lag and the time of formation of the discharge. In gases a lag of the breakdown of 10^{-4} sec and more has been observed [1-3]. In solid dielectrics, the lag of the breakdown is considerably less $(1\text{-}8) \cdot 10^{-8}$ sec [4, 5]. With such small exposures, in solid dielectrics an increase in the electrical strength has been observed [6]. While in gases a considerable part of the lag time of the breakdown consists of the statistical lag, and only with strong ionization of the spark gap will the lag time consist only of the time of formation of the discharge, in solids the lag time consists mainly of the formation time of the discharge.

Data on the lag time of two-phase dielectrics consisting of a solid body and a gas are of interest. A representative dielectric of this type is pressed lead azide, consisting of crystalline lead azide and air, which is investigated in the present work. A study of the breakdown of lead azide and an explanation of its mechanism is also of interest for a study of its sensitivity to an electric spark.

Method of Experiment

Powdered lead azide with a crystal size of 1-3μ was pressed between steel electrodes to a density of 2.8 g/cm^3. Under these circumstances, in the solid dielectric a porosity of 0.4 was set up (40% of the

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